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AEGIS SPY-1 RADAR 5-V/60-A SWITCHING-MODE POWER SUPPLY STABILITY ANALYSIS

A stability analysis employing state-space averaging techniques for converter modeling

RE Hammond

22 January 1979

Interim Report for Period 1 June — 1 October 1978

Prepared for
Naval Sea Systems Command, PMS 400B

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State of the art techniques for modeling switching regulator power supplies are applied to the AEGIS SPY-1 Radar 5-volt/60-ampere power supply. The results of the modeling and subsequent analysis are used to predict the power supply's basic stability and performance characteristics.

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OBJECTIVE

Apply recent advances in the modeling of switching regulator power supplies to the AEGIS SPY-1 Radar 5-volt/60-ampere power supply. Analyze this model with classical control theory techniques and make conclusions regarding the stability and performance characteristics of this power converter.

RESULTS

1. The AEGIS 5-volt/60-ampere power supply is modeled by applying Middlebrook's state-space averaged canonical model to the nonlinear output power stage. The complete control theory model including feedback loops was analyzed and found to be stable. The regulator's open-loop gain has a high dc value and an ample bandwidth, thus, good transient response is predicted.

2. Also modeled and analyzed are the line transmission characteristic and output impedance. The line transmission characteristic shows the power supply to have very good attenuation of input line variations. This is due partly to the feedforward employed in the supply. The output impedance determined by the model is more than acceptable for this regulator.

RECOMMENDATIONS

Confirm the analytical results for the power supply with experimental measurements and testing on an actual power supply. The testing should include step load change testing, line transmission testing, and experimental determination of tne open-loop gain. Such measurements and tests will verify both the stability and performance of the power supply.

CONTENTS

INTRODUCTION

Like other electronic system components, power supplies are being made smaller, lighter, and more efficient. This means that traditional straightforward linear power supplies are being replaced with smaller, more efficient switching regulator power supplies. By their very nonlinear switching nature, these power supplies are difficult to analyze. However, it is necessary to be able to accurately model these supplies to be able to stabilize their control loops and to meet performance specifications. Thus, a problem has existed with switching-mode power supplies and their design has been considered something of a black art.

Recently, techniques have been developed that allow modeling of the nonlinear power stage of a switching regulator with comparative ease. Approaches such as those developed by Middlebrook, Prajoux, Yu, and others permit the use of standard control theory and circuit analysis in analyzing regulator stability and performance characteristics (ref. 1–7).

The linearized canonical model of Middlebrook is employed in this analysis. This model is derived by averaging the state-space equations for the converter in either the continuous or discontinuous inductor current modes. It has been shown that this model produces valid results for frequencies up to almost half the regulator's switching frequency. The model is useful for predicting performance and stability characteristics at steady state and steady state with small perturbations. The model has several extremely useful and salient features. First, it represents the most concise, least complicated approach to modeling switching regulators yet developed. Next, an attempt has been made through the canonical model to retain physical insight and to provide a means of comparison among different types of converters. Finally, the model developed is completely linear. Thus, the complete field of linear control theory is made applicable.

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1. RD Middlebrook and Slobodan C 'uk, A General Unified Approach to Modeling Switching-Converter Power Stages, IEEE Power Electronics Specialists Conference, 1976 Record, 18–34 (IEEE Publication 76CH1084-3 AES); also International J of Electronics, vol 42, no 6, p 521–550, June 1977
2. Slobodan C 'uk, Modeling, Analysis, and Design of Switching Convertors, PhD thesis, California Institute of Technology, November 1976
3. RD Middlebrook and Slobodan C 'uk, Modeling and Analysis Methods for Dc-to-Dc Switching Converters, invited review paper, IEEE International Semiconductor Power Converter Conference, 1977 Record, p 90–111 (IEEE Publication 77CH1183-31A)
4. RD Middlebrook, Input Filter Considerations in Design and Application of Switching Regulators, IEEE Industry Applications Society Annual Meeting, 1976 Records, p 366–382 (IEEE Publication 76CH1122-1-IA)
5. RD Middlebrook, Design Techniques for Preventing Input-Filter Oscillations in Switching-Mode Regulators, Proc Fifth National Solid State Power Conversion Conference (POWERCON 5), p A3.1–A3.16, May 1978
6. A Capel, JG Ferrante, and R Prajoux, State Space Stability Analysis of Multi-Loop PWM Controlled DC/DC Regulators in Heavy and Light Mode, IEEE 1975 Power Electronics Specialists Conference Record, June 1975
7. HA Owen, A Capel, and JG Ferrante, Simulation and Analysis Methods for Sampled Power Electronic Systems, IEEE 1976 Power Electronics Specialists Conference Record, June 1976

Figure 1 shows the basic power stages for the three most common types of converters. Also shown is the canonical model to which all three converters are reduced. Even though schematically all three converters have the same canonical model, the values for the model circuit elements are different, as shown in table 1.

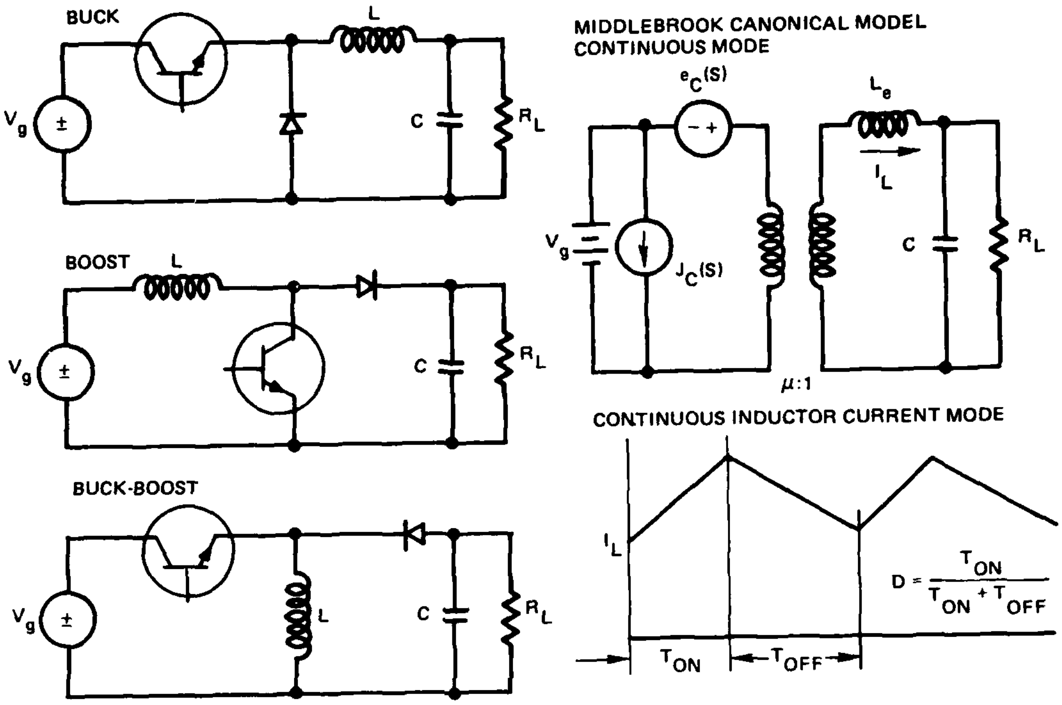

Figure 1. Common converters and their equivalent circuit.

Table 1. Canonical model element values.

TYPE	u(D)	E	$f_1(s)$	J	$f_2(s)$	Le
Buck	$\frac{1}{D}$	$\frac{V}{D^2}$	1	$\frac{V}{R}$	1	L
Boost	1-D	V	$1-S\frac{Le}{R}$	$\frac{V}{(1-D)^2R}$	1	$\frac{L}{(1-D)^2}$
Buck-boost	$\frac{1-D}{D}$	$\frac{-V}{D^2}$	$1-S\frac{DLe}{R}$	$\frac{-V}{(1-D)^2R}$	1	$\frac{L}{(1-D)^2}$

Several features are pertinent as an introduction to the AEGIS power supply. A basic schematic which shows the components relevant to the control loops is included in *appendix A*. The AEGIS power supply is a fixed-frequency (30 kHz), duty-ratio-modulated buck switching regulator. Feedforward is utilized to improve the line transmission characteristic and to attenuate input line disturbances. Two feedback loops are employed to produce a volt-ampere characteristic which permits efficient paralleling of power supplies for load sharing and reliability. The operation of the two control loops can be explained briefly as follows. The first operational amplifier, A1, is a voltage comparator with an internal reference voltage. The output voltage is sampled and compared to this reference voltage. The result of this comparison becomes the input to A2, which is used for voltage loop frequency shaping. A3 is a differential amplifier which is used to compare a current sample voltage with the output of A2. Finally, A4 is employed in a filter to remove switching frequency noise. A4's output is the control voltage input to the regulator's pulse width modulator. The overall operation of the two loops can be described as a *voltage-controlled current loop*.

Figure 2 is a photograph of the AEGIS 5-volt/60-ampere power supply.

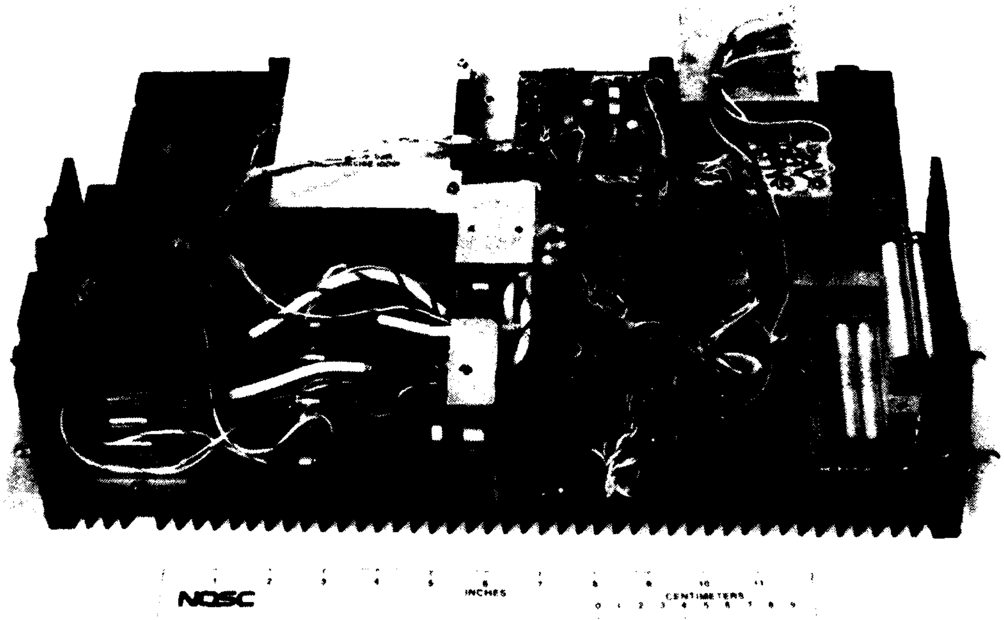

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Figure 2. AEGIS 5-volt/60-ampere power supply.

PROCEDURE

Utilization of the Middlebrook modeling techniques centers around replacement of the power stage of the converter with the canonical model. It must be determined whether the converter is operating in the continuous or discontinuous inductor current mode, as this determines which of the two general canonical models is to be employed. Then in a general sense it is necessary to determine the number of feedback loops and whether feed-forward exists in the converter. Symbols are assigned for various transfer functions, and block reduction and algebraic manipulation are performed to find the open-loop gain, line transmission characteristic, and output impedance. It is important that the transfer function between the reference voltage and the output voltage be found in determining the open-loop gain (T) for stability considerations.

The next phase in the analysis involves actual determination of individual transfer functions. Operational amplifiers in the feedback loops often present little problem in determining their transfer function. However, two aspects of the transfer function of operational amplifiers deserve special mention. First, the compensation or dominant pole roll-off of the amplifier gain must be investigated in general. Situations in which the amplifier develops high gain can be expected to produce a pole in the amplifier transfer function which must be included. Also, in situations in which a filter exists as an amplifier's input impedance, care must be exercised in ascertaining the correct impedance for amplifier gain. Reference to the derivation of the equations for operational amplifier gain may be helpful in this case. In addition, some decision should be made as to an upper limit for poles and zeros to be included in the analysis. In this analysis, poles and zeros breaking at past one-half the switching frequency are neglected. This decision is based upon two facts. First, the loop gain of most well designed switching regulators will cross 0 dB at around one-tenth of the switching frequency. Second, the linearized converter model is accurate only to one-half the switching frequency. Finally, linearized transfer functions for the modulator and feedforward should be found. Since the canonical model is a linearized model around an operating point (duty ratio), finding additional linearized transfer functions does not further restrict use of the model.

Once all the required transfer functions have been obtained, the analysis of the model is performed. Numerous approaches are available for the analysis of linear control theory models. Root locus, Nyquist analysis, Nichols Chart analysis, and Bode analysis are all viable approaches. Early on in the analysis the conclusion becomes apparent that some computer aid is necessary. This is true, for example, if a two-section output filter is used, or if multiple control loops are employed, as systems above third or fourth order arise. For these systems, computers represent the only feasible way to factor these transfer functions and find the required roots. Beyond this basic and often very necessary capability, a wide range of computer tools exists. Mid-size computers often have a complex arithmetic capability which will allow solution of virtually any quantity with the appropriate programming. Finally, there are two major areas of programs available to solve the types of problems presented by the analysis of the complete model. There are linear control theory programs which allow complete analysis once the appropriate transfer functions are entered. Then, too, there are basically circuit analysis programs such as Spice and Sceptre, which allow analysis of the model from a circuit approach.

RESULTS AND CONCLUSIONS

Figure 3 is a Bode plot of the open loop gain (T) of the model. A number of quantities are important to inspect on this graph. Of first concern is the stability of the converter. The phase and gain margins measure the converter's stability and in this case are a very adequate 40 degrees and 13 dB respectively. It is also important for the open loop gain to have a high DC value and a good bandwidth. Both of these quantities as shown are quite acceptable for the Aegis 5-volt/60-ampere converter.

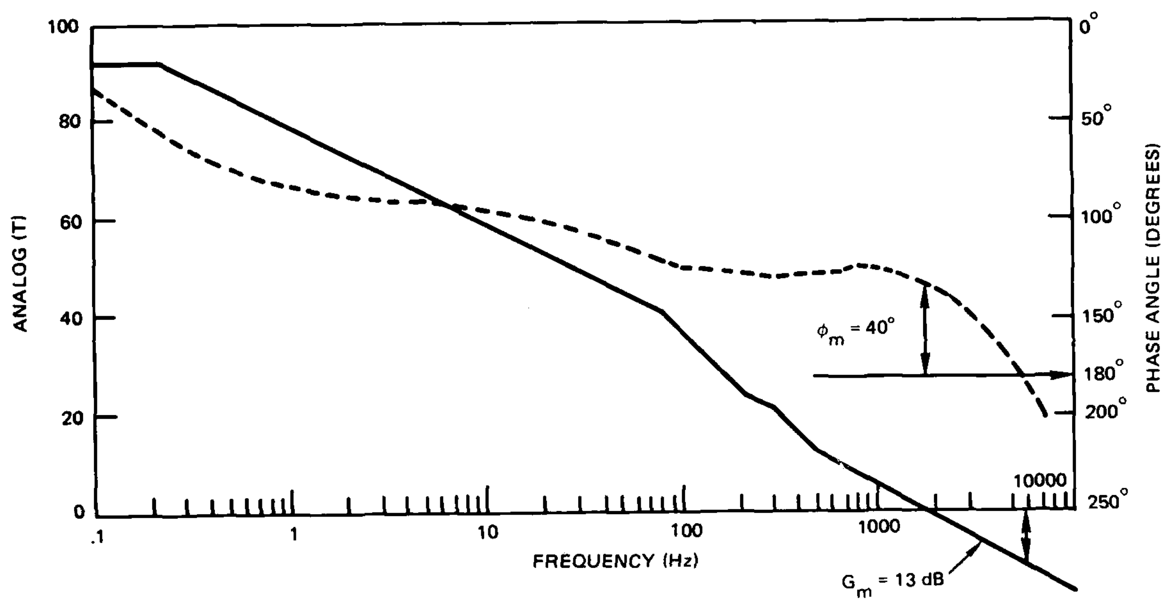

Figure 3. Loop gain (T).

Figure 4 is a Bode plot of the line transmission characteristic (F). This quantity indicates how much attenuation of the magnitude of input line voltage disturbances the power converter is able to supply as a function of frequency. This quantity is of extreme importance aboard Navy ships, as the voltage buses the power converters draw their power from are often subjected to extreme voltage transients. If the power converter is not able to sufficiently attenuate these transients, logic states may change in digital loads and, depending on the magnitude of the transient, damage to the load can occur. Figure 4 indicates the converter has an excellent line transmission characteristic as input line disturbances are attenuated by at least 59 dB.

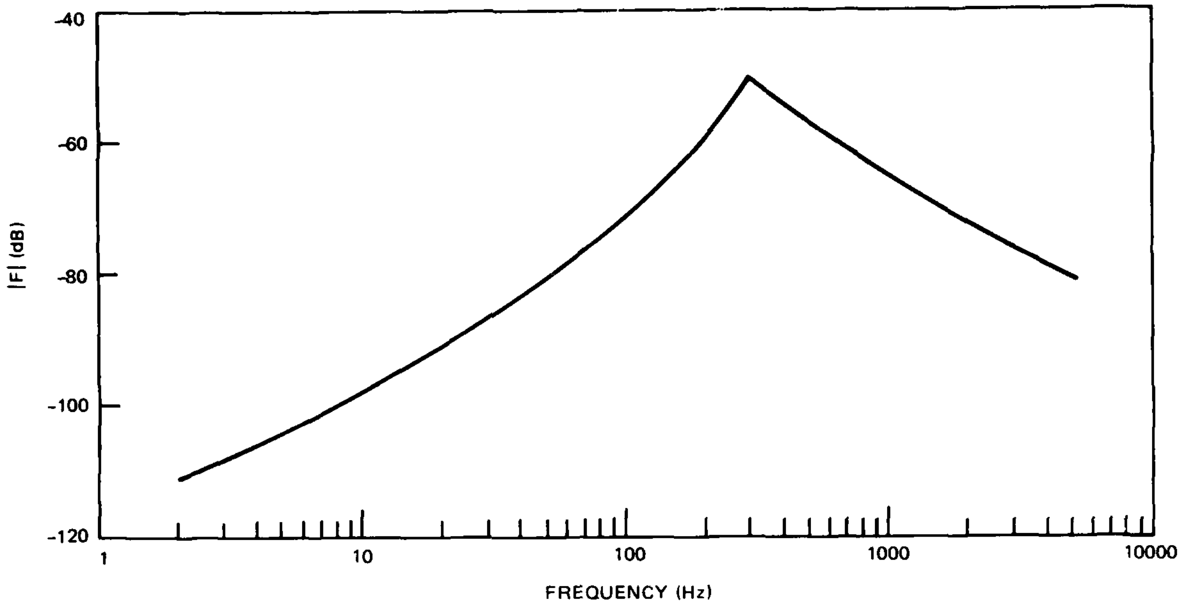

Figure 4. Line transmission characteristic (F).

Figure 5 is a Bode plot of the converter output impedance. This quantity is important, as it is a measure of the converter's ability to maintain its output voltage despite load changes. The smaller the output impedance at a particular frequency, the smaller the noise component on the output voltage at that frequency during load variations. The plot of output impedance shown in figure 5 indicates that converter output impedance is excellent over the entire frequency range.

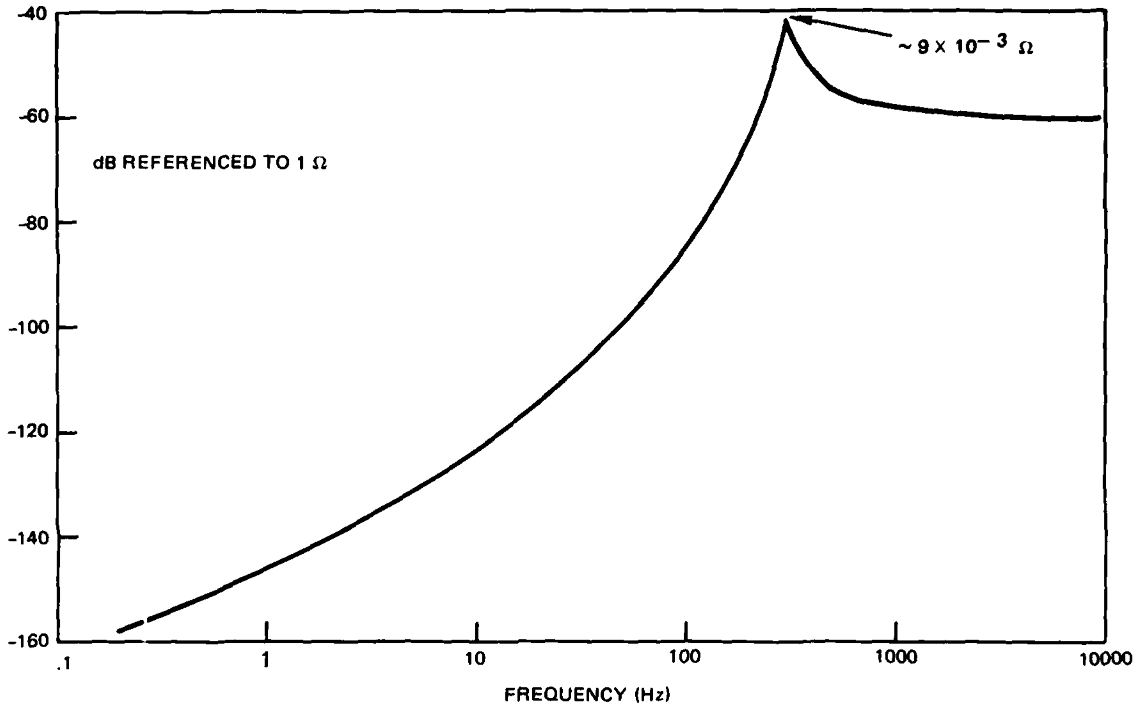

Figure 5. Output impedance (Z_0).

REFERENCES

1. RD Middlebrook and Slobodan C'uk, A General Unified Approach to Modeling Switching-Converter Power Stages, IEEE Power Electronics Specialists Conference, 1976 Record, p 18–34 (IEEE Publication 76CH1084-3 AES); also International J of Electronics, vol 42, no 6, 521–550, June 1977

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7. HA Owen, A Capel, and JG Ferrante, Simulation and Analysis Methods for Sampled Power Electronic Systems, IEEE 1976 Power Electronics Specialists Conference Record, June 1976

APPENDIX A: MODELING THE CONVERTER

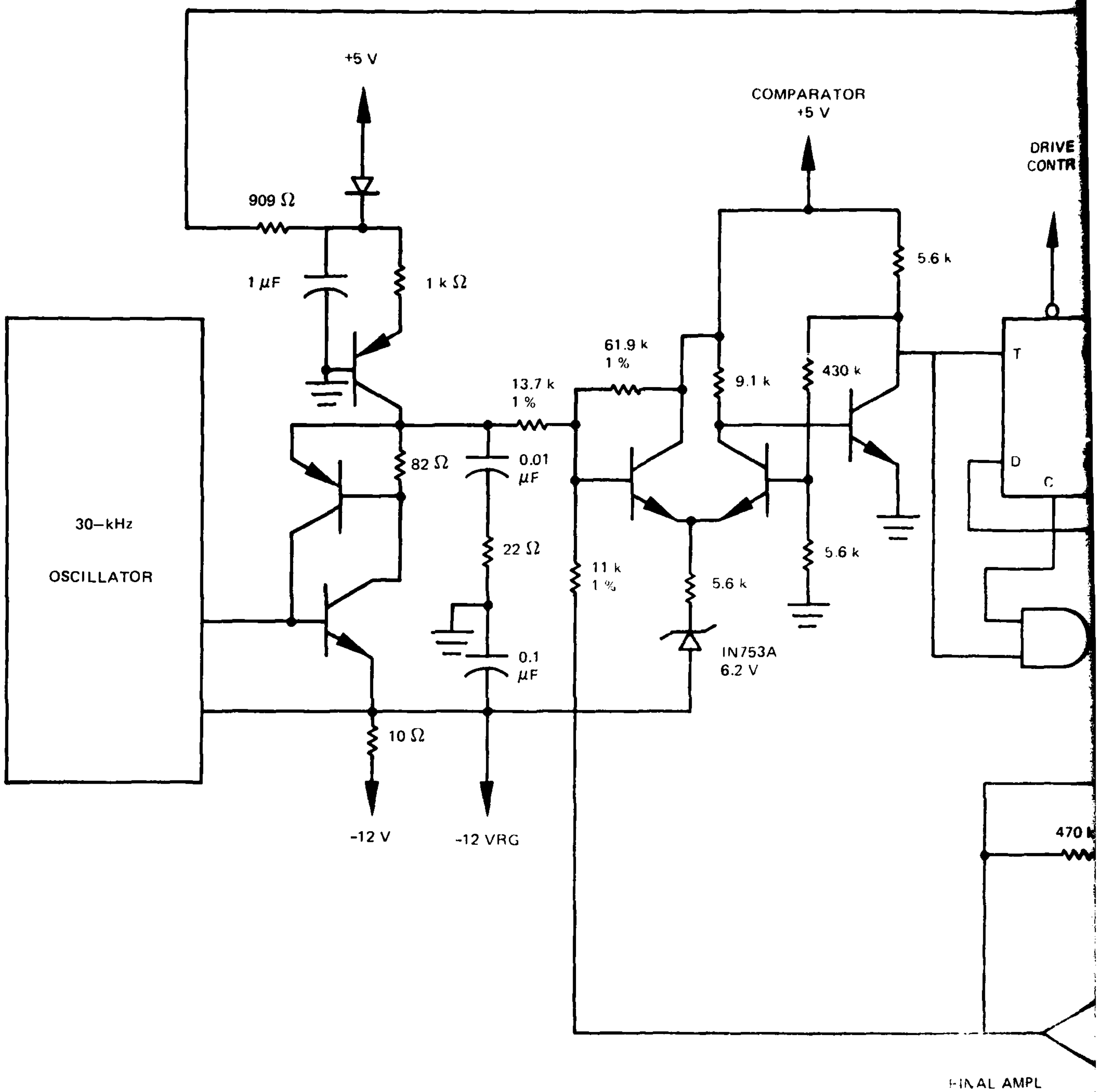
Converter schematic – (simplified to show only feedback loop components)

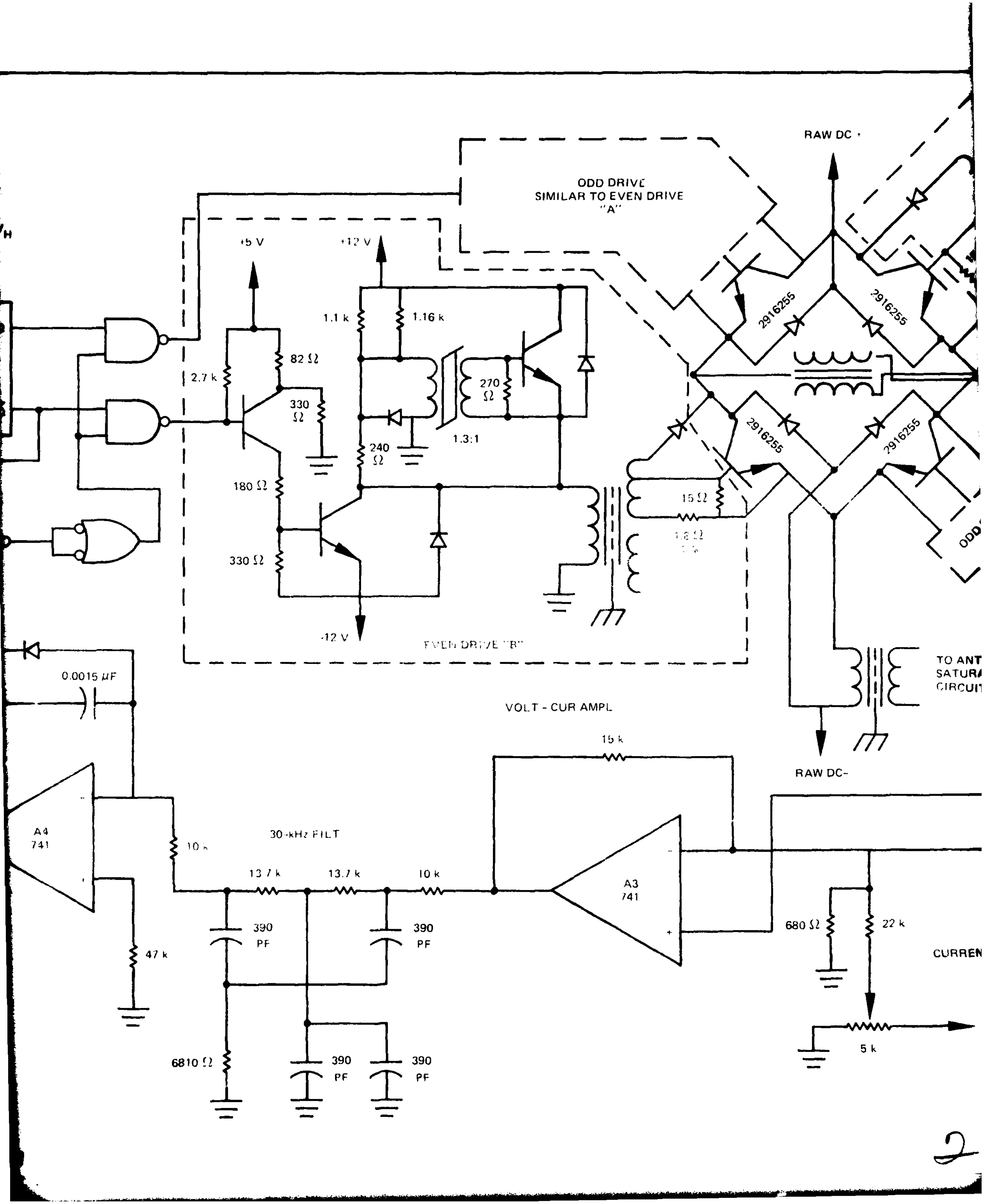
General canonical model

Open-loop gain

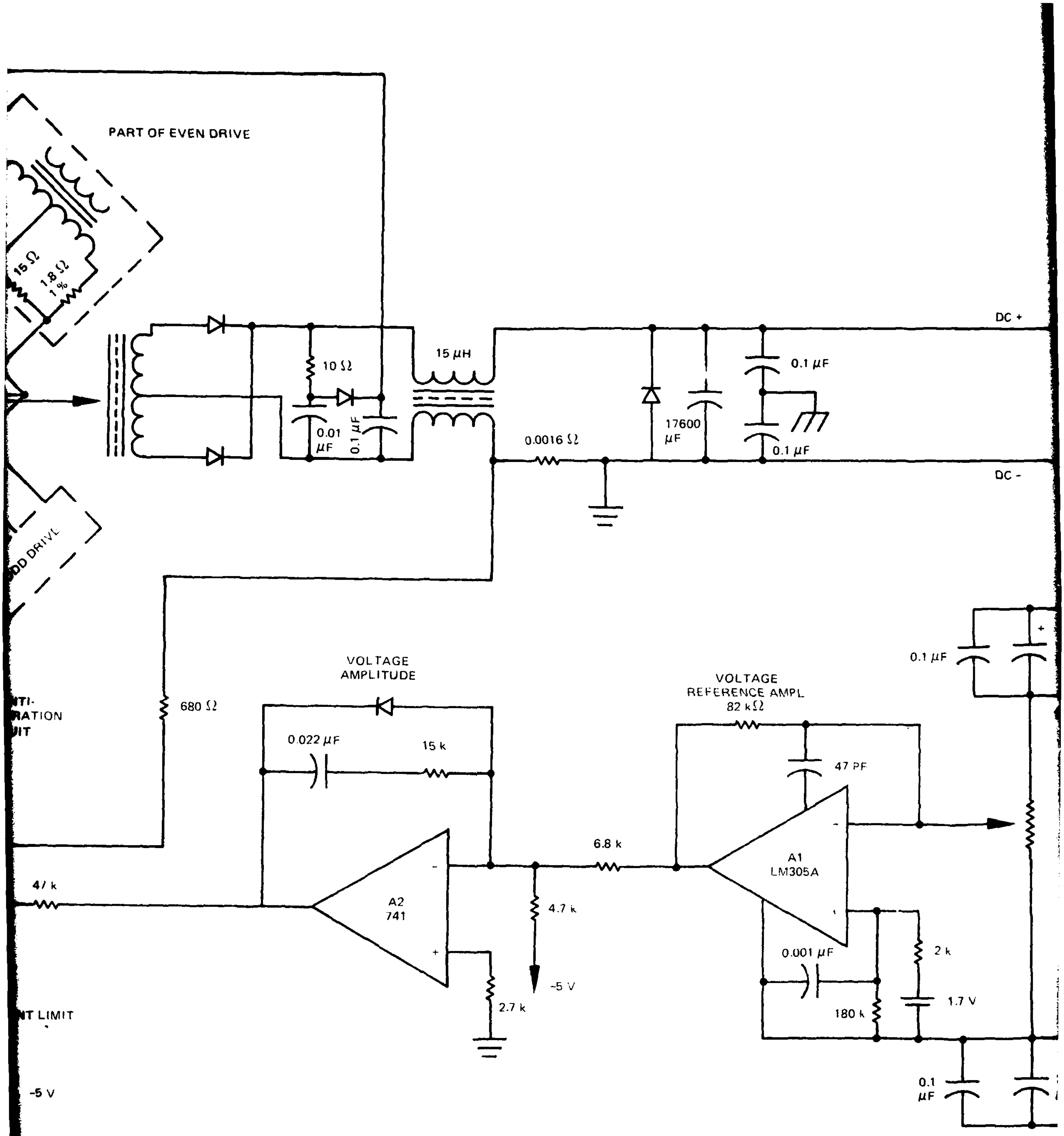
Line transmission characteristic

Output impedance

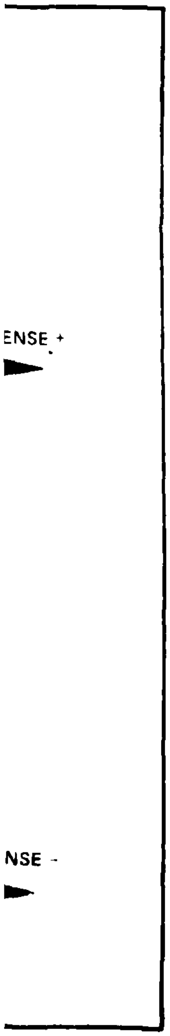

+5 V
COMPARATOR
+5 V
DRIVE
CONTR
909 Ω
1 μF
1 k Ω
5.6 k
61.9 k
1 %
13.7 k
1 %
9.1 k
430 k
T
D
C
82 Ω
0.01
μF
30–kHz
OSCILLATOR
22 Ω
11 k
1 %
5.6 k
5.6 k
IN753A
6.2 V
0.1
μF
10 Ω
-12 V
-12 VRG
470 k
FINAL AMPL



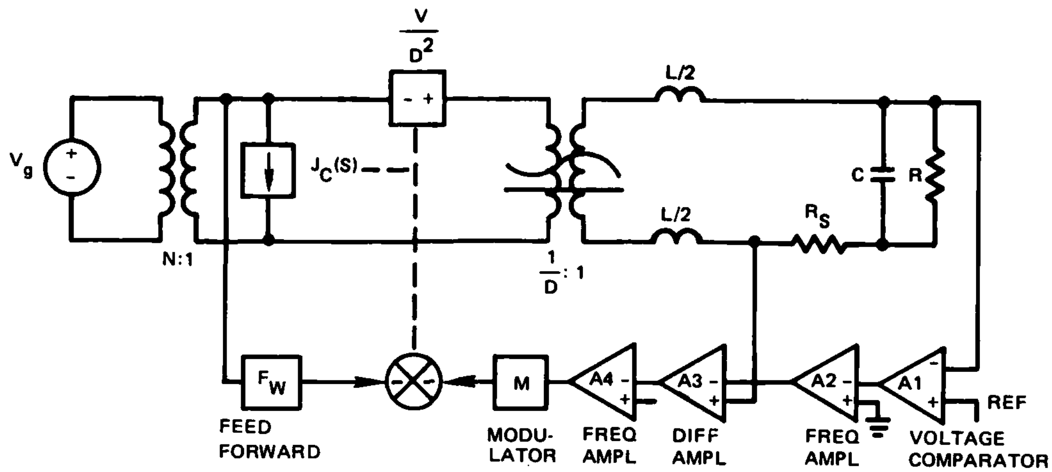
RAW DC +
ODD DRIVE
SIMILAR TO EVEN DRIVE
"A"
+5 V
+12 V
1.1 k
1.16 k
82 Ω
2.7 k
330 Ω
270 Ω
240 Ω
1.3:1
180 Ω
330 Ω
15 Ω
-12 V
EVEN DRIVE "B"
2916255
2916255
2916255
2916255
TO ANT
SATUR
CIRCUI
0.0015 μF
VOLT - CUR AMPL
15 k
RAW DC-
A4
741
10 k
30-kHz FILT
13.7 k
13.7 k
10 k
A3
741
680 Ω
22 k
390 PF
390 PF
47 k
6810 Ω
390 PF
390 PF
CURREN
5 k
ODD



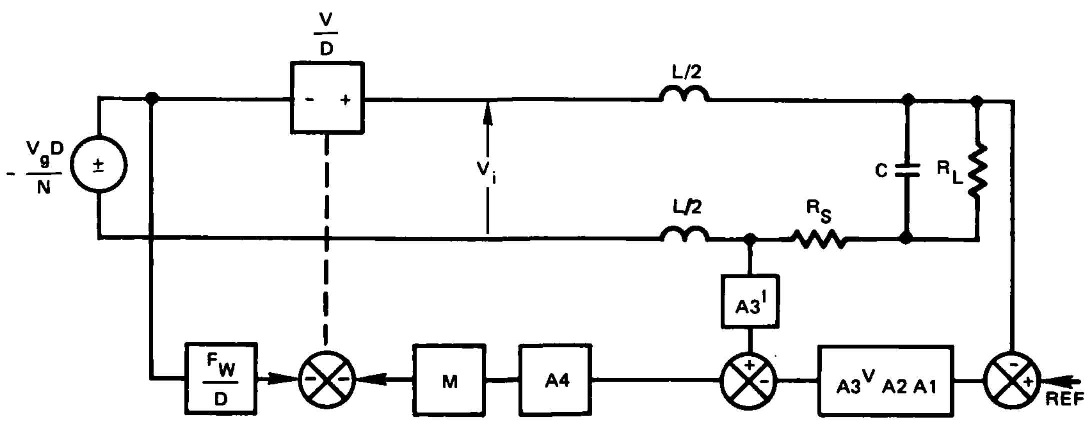
AEGIS 5-V/60-A Control Loop


ENSE +
NSE -

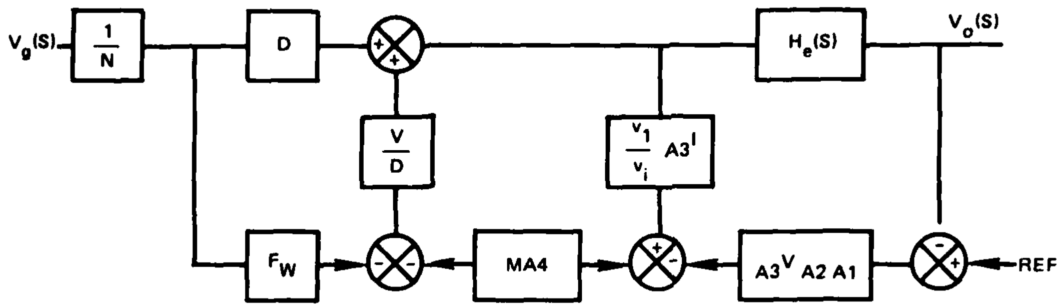
GENERAL CANONICAL MODEL



This model is further simplified to:

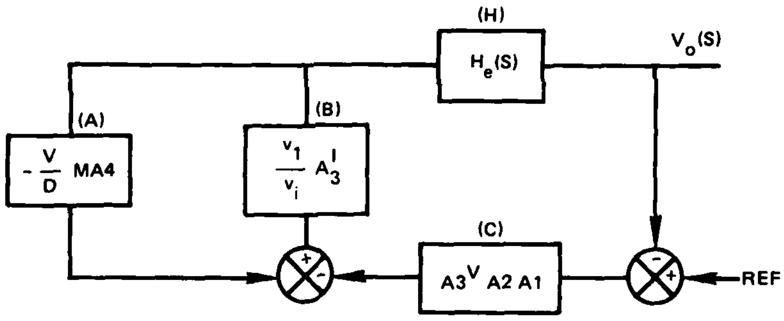


In simplified block diagram form:



OPEN-LOOP GAIN (T)

The above block diagram is further simplified to:



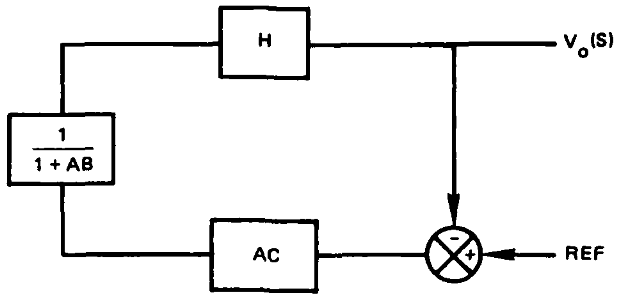
$$A = -\frac{V}{D}\, MA4$$

$$B = \frac{v_1}{V_i}\, A3^I$$

$$C = A3^V\ A2\ A1$$

$$H = He(S)$$

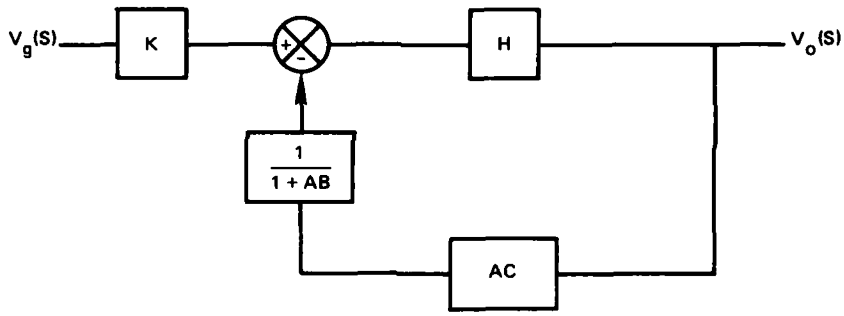
which is further simplified to:



from which $T = \dfrac{ACH}{1 + AB}$.

LINE TRANSMISSION CHARACTERISTIC (F)

The original block diagram is simplified to:



where $K = \dfrac{1}{N}\left(D - F_W \dfrac{V}{D}\right)$

from which

$$F = \frac{V_o(S)}{V_g(S)} = \frac{KH}{1 + \dfrac{ACH}{1 + AB}} = \frac{KH}{1 + T}$$

OUTPUT IMPEDANCE (Z_o)

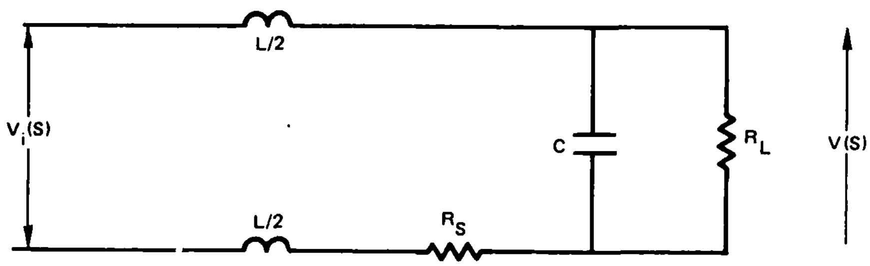
Z_{oe} = output impedance of output power averaging filter

Z_o = output impedance of converter with closed feedback loop

It is known by classical control theory $Z_o = \dfrac{Z_{oe}}{1 + T}$.

APPENDIX B: MODELING OF CONVERTER COMPONENTS

OUTPUT FILTER VOLTAGE TRANSFER FUNCTION



$$H_e(s) = \frac{V(s)}{V_i(s)} = \frac{1}{LCs^2 + s\left(\frac{L}{R_L} + R_sC\right) + \left(1 + \frac{R_s}{R_L}\right)}$$

For AEGIS 5-V/60-A supply:

$L = 1.5 \times 10^{-5}$ H

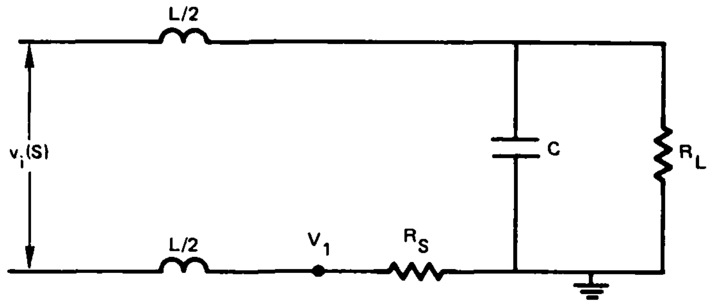
$R_s = 0.0016\ \Omega$

$R_L = 5.0/60\ \Omega = 0.08333$

$C = 1.76 \times 10^{-2}$ F

$$H_e(s) = \frac{3.79 \times 10^6}{(s + 394 + 1.92 \times 10^3_j)(s + 394 - 1.92 \times 10^3_j)}$$

OUTPUT FILTER CURRENT TRANSFER FUNCTION



$$\frac{V_1(s)}{V_i(s)} = \frac{R_s R_L C s + R_s}{s^2 L R_L C + s(L + R_s R_L C) + (R_L + R_s)}$$

For AEGIS 5-V/60-A supply

R_s = 0.0016 Ω

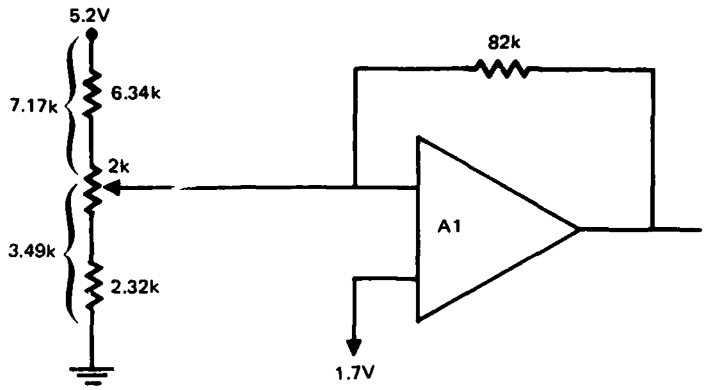
R_L = 5/60 Ω = 0.08333

C = 1.76×10^{-2}F

L = 1.5×10^{-5}H

$$\frac{V_1(s)}{V_i(s)} = \frac{+1.07 \times 10^2 (s + 6.82 \times 10^2)}{(s + 394 + 1.92 \times 10^3_j)(s + 394 - 1.92 \times 10^3_j)}$$

VOLTAGE AMPLIFIER A1



$3.49 \parallel 7.17k = 2.35\ k\Omega$

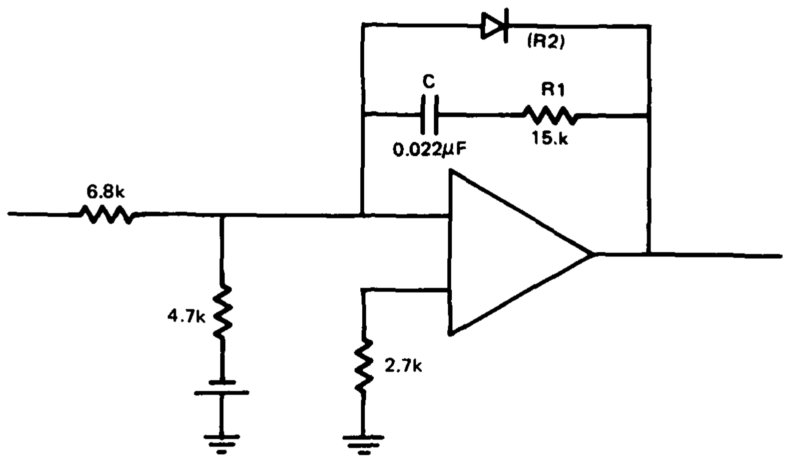
$$A = \frac{-Z_{fb}}{Z_{in}} = \frac{-82k}{2.35k} = -34.9$$

with the input voltage divider included

$$A1 = -34.9 \times \frac{3.49k}{7.17k + 3.49k} = -11.42$$

Assuming a gain bandwidth product of 900 kHz, the LM105's dominant pole breaks at past one-half the switching frequency and is neglected for simplicity.

VOLTAGE AMPLIFIER A2



$$R_{11} = \frac{(6.8)\,(4.7)k}{6.8 + 4.7} = 2.78k$$

if the diode is modeled as a large resistance of a reverse biased diode.

$$R_2 = 45 \times 10^6 \Omega$$

$$Z_{fb} = \frac{R1R2}{R1 + R2} \; \frac{\left(S + \frac{1}{R1C}\right)}{\left(S + \frac{1}{C(R1 + R2)}\right)}$$

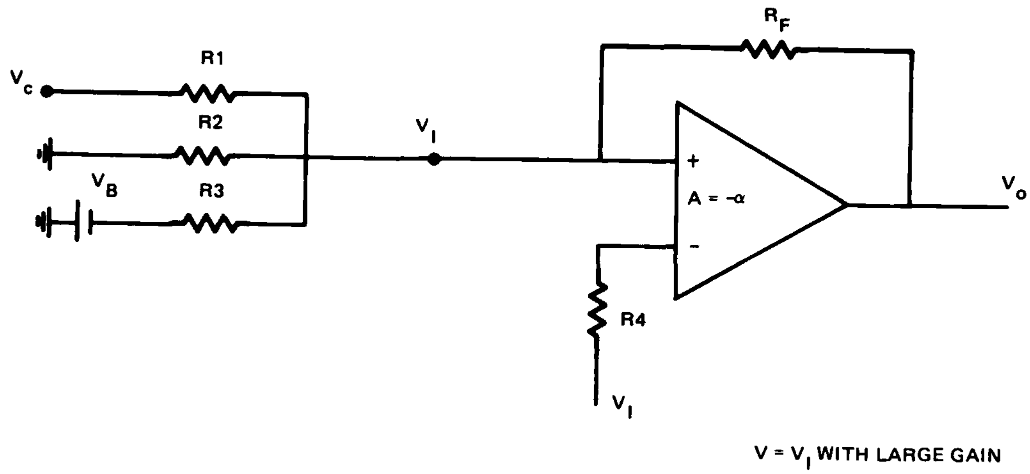
$$A2 = \frac{-Z_{fb}}{R_{11}} = -\frac{5.4\,(S + 3.03 \times 10^3)}{(S + 1)}$$

With the input voltage divider included

$$A2 = \frac{4.7k}{4.7k + 6.8k} \; \frac{(S + 3.03 \times 10^3)}{(S + 1)} = \frac{2.2\,(S + 3.03 \times 10^3)}{(S + 1)}$$

Amplifier's dominant pole breaks at 185 MHz, which is high enough to be neglected.

DIFFERENTIAL AMPLIFIER A3



KCL

$$\frac{V_c-V_I}{R_1} + \frac{-V_I}{R_2} + \frac{-V_B-V_I}{R_3} + \frac{V_o-V_I}{R_f} = 0$$

For AEGIS 5-V/60-A supply

$R_1 = 47k$

$R_2 = 0.68k$

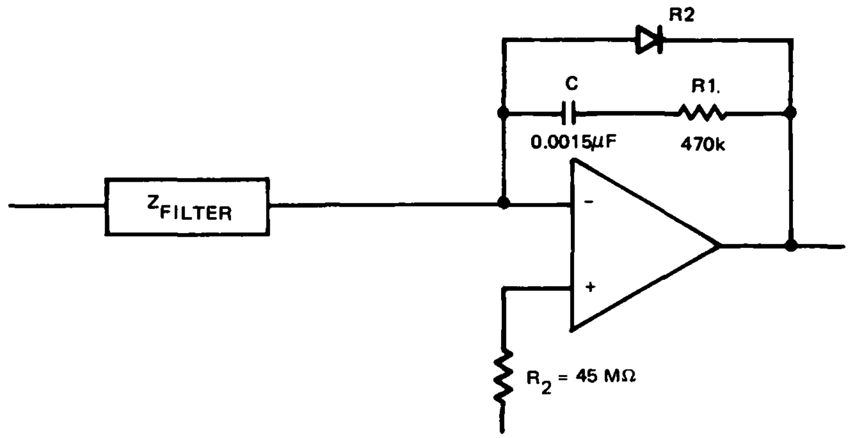
$R_3 = 23k$

$R_f = 15k$

$$V_o = 24V_1 + 0.65V_B - 0.32V_c$$

With a gain of 24, the dominant pole breaks at 2.6×10^5 r/s, which is high enough to be neglected.

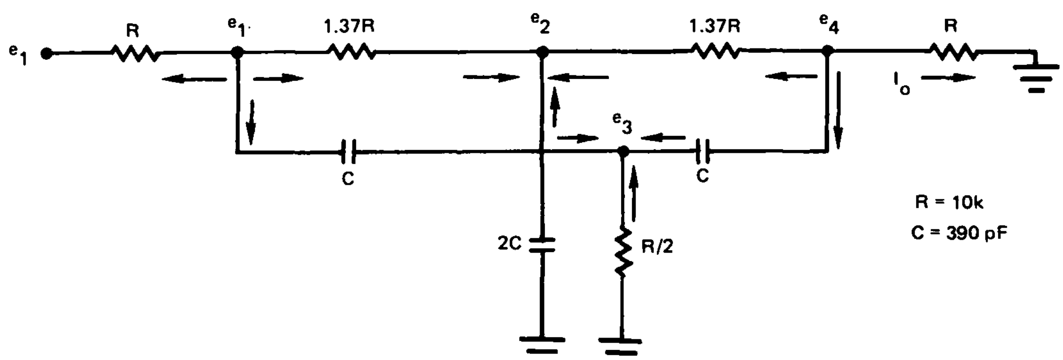
FINAL AMPLIFIER A4



Feedback Impedance (Diode Reverse Biased)

$$Z_{fb} = \frac{R1\ R2}{R1 + R2} \frac{S + \frac{1}{R1C}}{S + \frac{1}{C(R1 + R2)}} = \frac{4.66 \times 10^5 (S + 1410)}{(S + 13.2)}$$

30-kHz FILTER IMPEDANCE



Kirchoff's Current Law is written at the four nodes

$$(1)\quad -\frac{e_1 - e_i}{R} - \frac{e_1 - e_2}{1.37R} - \frac{e_1 - e_3}{1/SC} = 0$$

$$(2)\quad \frac{e_2 - e_1}{1.37R} + \frac{e_2 - e_4}{1.37R} + \frac{e_2}{1/2SC} = 0$$

$$(3)\quad \frac{e_3 - e_1}{1/SC} + \frac{e_3 - e_4}{1/SC} + \frac{e_3}{R/2} = 0$$

$$(4)\quad -\frac{e_4 - e_2}{1.37R} - \frac{e_4 - e_3}{1/SC} - \frac{e_4}{R} = 0$$

These equations are put into standard form and with matrix algebra the quantity of $Z = \frac{e_i}{I_o}$ is solved for.

$$Z_{filter} = \frac{e_i}{I_0} = \frac{Re_i}{e_4}$$

$$= 2\,\frac{S^2 1.37R^2C^2(3.37R^2) + SC(9.36R^2) + 2.37R}{S^2 1.88R^2C^2 + 1}$$

For AEGIS 5 V/60 A, R = 10k C = 390 pF.

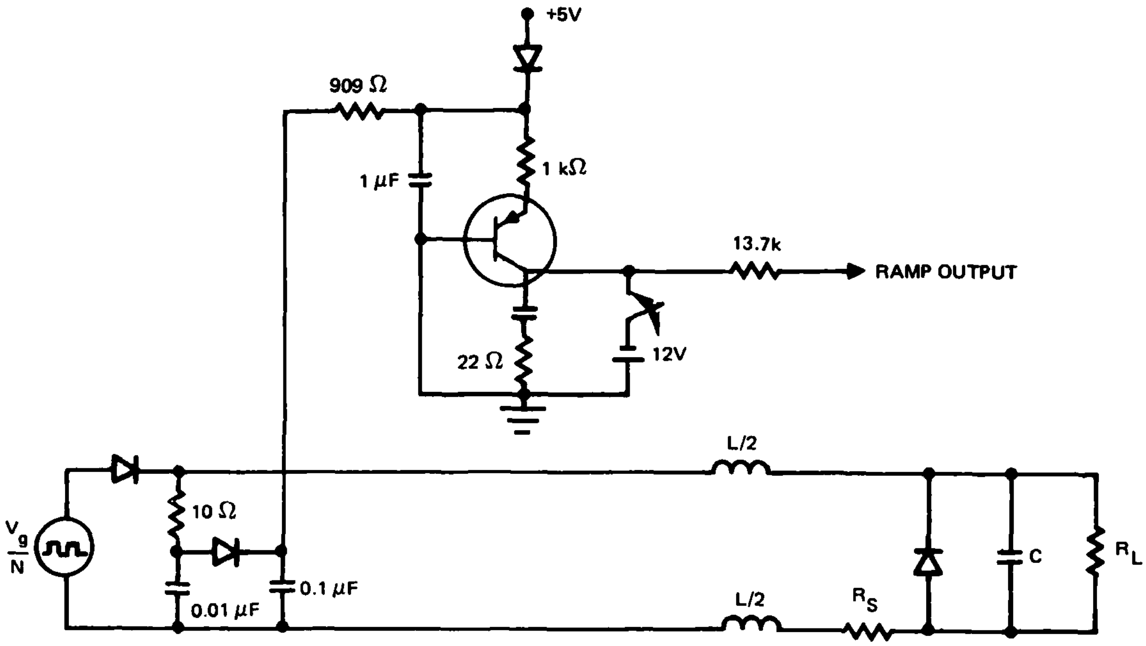
$$Z_{filter} = \frac{4.9 \times 10^4\,(S + 7.33 \times 10^4)(S + 4.6 \times 10^5)}{(S + 1.87 \times 10^5 j)(S - 1.87 \times 10^5 j)}$$

After removing terms coming in past one-half the switching freq $\omega > 10^5$ r/s

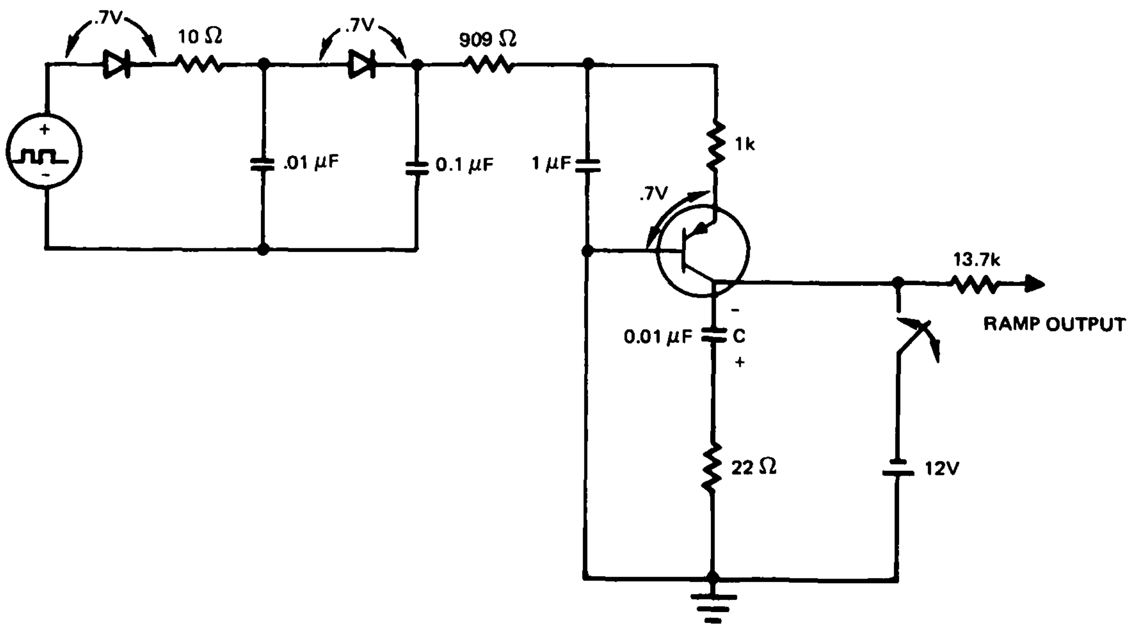
$$Z_{filter} = 0.644(S + 7.33 \times 10^4)$$

$$A4 = -\frac{Z_{fb}}{Z_{filter}} = \frac{7.24 \times 10^5\,(S + 1410)}{(S + 13.2)(S + 7.33 \times 10^4)}$$

MODULATOR AND FEEDFORWARD



This diagram is simplified to:



After the transistor switches close then open, the voltage on the capacitor C starts out at -12 V minus the drop across the transistor switches. The capacitor then charges as the next input pulse is received.

Input pulse magnitude $= \frac{V_g}{N} = \frac{155}{11.4} = 13.6$ V

Nominal charging current $I_c = \frac{13.6 - 3(0.7)}{10 + 909 + 1000} = 6.0$ mA

General charging current $I_c = \frac{\frac{V_g}{11.4} - 2.1\text{V}}{1.919 \times 10^3 \Omega} = 4.57 \times 10^{-5} (V_g - 24)$

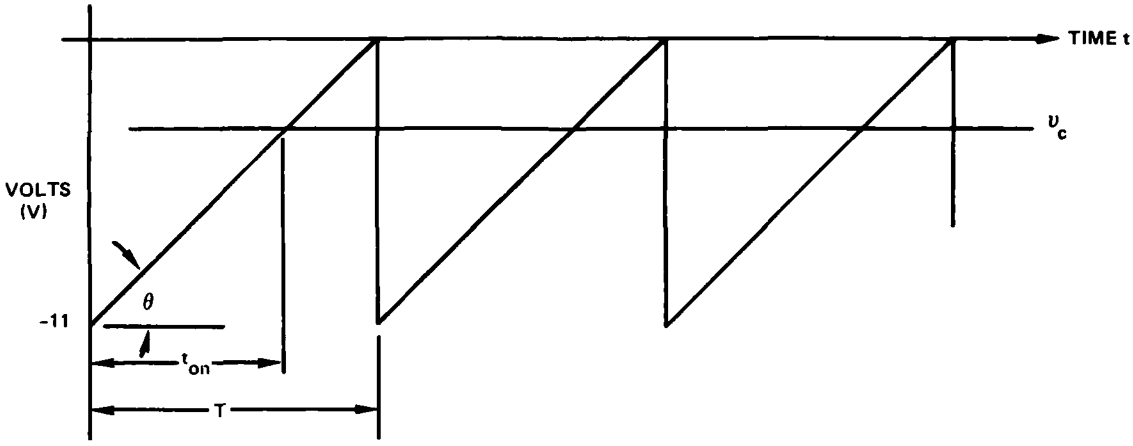
Nominal duty ratio $D = \frac{5.2}{13.6 - 0.7} = 0.4$

Nominal charging rate $\frac{V}{T} = \frac{I}{C} = \frac{6.0 \times 10^{-3}}{10^{-8}} = 0.60$ V/μs

Nominal control voltage (D = 0.40)

$$v_c = t_{on} \times \frac{V}{T} = (0.4)(33\ \mu\text{s})(0.60\ \text{V}/\mu\text{s}) = 7.9\text{V}$$

RAMP VOLTAGE



$$D = \frac{t_{on}}{T} = \frac{v_c}{T_{\tan\theta}} \quad , \quad \tan\theta = \frac{I_c}{C}$$

$$= \frac{v_c}{T\frac{I_c}{C}} = \frac{Cv_c}{T\,I_c} \;, \quad I_c = 4.57 \times 10^{-5}(V_g - 24)$$

$$= \frac{Cv_c}{T(4.57 \times 10^{-5})(v_g - 24)} = \frac{(10^{-8})v_c}{(33 \times 10^{-6})(5 \times 10^{-5})(v_g - 24)} = \frac{6.63(v_c)}{(v_g - 24)}$$

$$\Delta D = \frac{\partial D}{\partial V_g}\,\Delta V_g + \frac{\partial D}{\partial V_c}\,\Delta V_c$$

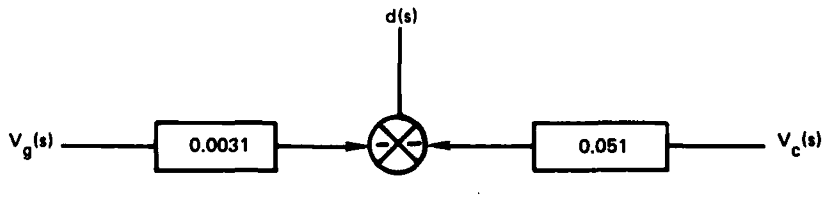
$$\Delta D = \frac{(-1)(6.63)v_c}{(v_g - 24)^2}\Delta V_g + \frac{6.63}{(v_g - 24)}\;\Delta V_c$$

$v_g = 155$ V

$v_c = 7.9$ V

$\Delta D = 0.0031\Delta V_g + 0.051\Delta V_c$

$d(s) = 0.0031\ V_g(s) + 0.051\ V_c(s)$



SUMMARY OF COMPONENT TRANSFER FUNCTIONS

$$H_e(s) = \frac{3.79 \times 10^6}{(S + 394 + 1920j)(S + 394 - 1920j)}$$

$$\frac{v_1}{v_i} = \frac{107(S + 682)}{(S + 394 + 1920J)(S + 394 - 1920j)}$$

$$A1 = 11.42$$

$$A2 = \frac{2.2\,(S + 3.03 \times 10^3)}{(S + 1)}$$

$A^{I}3 = 24$ $\qquad$ $A3^{V} = 0.32$

$$A4 = \frac{7.24 \times 10^5 (S + 1.41 \times 10^3)}{(S + 13.2)(S + 7.33 \times 10^4)}$$

$M = 0.051$ $\qquad$ $F_W = 0.0031$

APPENDIX C: ANALYSIS OF THE MODEL

Summary of defined values

Calculation of open-loop gain (T)

Calculation of line transmission characteristic (F)

Calculation of output filter output impedance (Z_o)

SUMMARY OF DEFINED VALUES

$$A = \frac{V}{D} \, M \, A4 = \frac{4.8 \times 10^5 (S + 1410)}{(S + 13.2)(S + 73300)}$$

$$B = \frac{v_i}{v_i} \, A3^I = \frac{2.57 \times 10^3 (S + 682)}{(S + 394 \pm 1920j)}$$

$$C = A1 \; A2 \; A3^V = \frac{8\,(S + 3030)}{(S + 1)}$$

$$H = H_e(s) = \frac{3.79 \times 10^6}{(S + 394 \pm 1920j)}$$

$$K = \frac{1}{N}\left(D - F_w \frac{V}{D}\right) = 0.032$$

CALCULATION OF OPEN-LOOP GAIN (T)

As previously derived

$$T = \frac{ACH}{1 + AB}$$

$$\frac{1}{1 + AB} = \frac{(S + 13.2)(S + 7.33 \times 10^4)(S + 394 \pm 1920j)}{(S + 540)(S + 1960)(S + 22600)(S + 48900)}$$

Thus

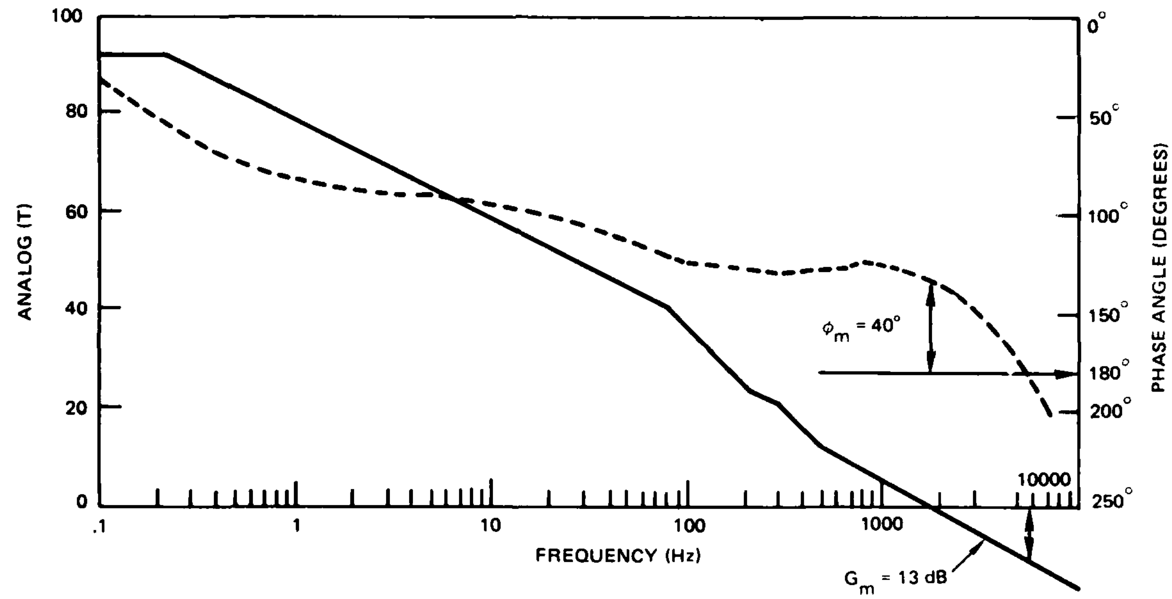
$$T = \frac{ACH}{1 + AB} = \frac{1.46 \times 10^{13}(S + 1410)(S + 3030)}{(S + 1)(S + 540)(S + 1960)(S + 22600)(S + 48900)}$$

A short program written in BASIC to compute phase and magnitude is shown on the following page. The program's output is plotted to display the Bode plot of the open-loop gain (T).

```
1 SELECT PRINT 215
2 PRINT
5 PRINT
10 PRINT "BODE PHASE AND MAGNITUDE PLOTS FOR T"
20 PRINT "DUTY RATIO=.4"
30 PRINT "FREQUENCY","20LOG T",,"PHASE ANGLE"
40 T=.1
60 FOR V=1 TO 7
70 FO=V*T
75 W=FO*6.2831853
80 X=W^2
90 A=SQR(X+1410^2)
100 A1=ARCTAN(W/1410)
110 B=SQR(X+3030^2)
120 B1=ARCTAN(W/3030)
130 C=SQR(X+1)
140 C1=ARCTAN(W)
150 D=SQR(X+540^2)
160 D1=ARCTAN(W/540)
170 E=SQR(X+1960^2)
180 E1=ARCTAN(W/1960)
190 F=SQR(X+22600^2)
200 F1=ARCTAN((W)/22600)
210 G=SQR(X+48900^2)
220 G1=ARCTAN(W/48900)
230 K=1.46E13
240 K1=57.296
250 Z=K1*(A1+B1-C1-D1-E1-F1-G1)
260 Y=8.6858896*LOG(K*A*B/(C*D*E*F*G))
270 PRINT FO,Y,,Z
280 NEXT V
290 T=T*10
300 IF FO<=60000 THEN 60
310 END
```

BODE PHASE AND MAGNITUDE PLOTS FOR T
DUTY RATIO=. 4

FREQUENCY	20LOG T	PHASE ANGLE
. 1	93. 09387354744	-32. 19198148563
. 2	90. 42418421575	-51. 58821105988
. 3	87. 95587001361	-62. 20340140162
. 4	85. 89580292908	-68. 50307788215
. 5	84. 17667958642	-72. 59323871276
. 6	82. 71682996023	-75. 44393295752
. 7	81. 45423612663	-77. 54068888008
1	78. 46618569391	-81. 45672379263
2	72. 52522469928	-86. 44923714202
3	69. 01597095308	-88. 46127408062
4	66. 51884228314	-89. 71797737142
5	64. 5783755457	-90. 67108524888
6	62. 99031112091	-91. 47167831108
7	61. 64536330704	-92. 18452345266
10	58. 52183005243	-94. 05493636598
20	52. 34927984786	-99. 31108889095
30	48. 58682385385	-103. 9566257328
40	45. 77667910864	-108. 1211466269
50	43. 47551188169	-111. 7983774736
60	41. 49540002709	-114. 992115921
70	39. 74188717257	-117. 7269322715
100	35. 39813164497	-123. 6016383811
200	26. 19898438214	-129. 7872837379
300	20. 8240140038	-129. 7531826316
400	17. 17594805983	-128. 8090035139
500	14. 47356804731	-127. 99490256
600	12. 35275931301	-127. 5387857447
700	10. 61664488819	-127. 4593475956
1000	6. 746849319322	-129. 0062596375
2000	-. 8258292318707	-142. 1163527927
3000	-5. 897535104928	-156. 7402148982
4000	-10. 05218551857	-169. 6324711317
5000	-13. 66626615102	-180. 5112860134
6000	-16. 89019917578	-189. 6276933584
7000	-19. 80641206328	-197. 2947857892
10000	-27. 18104018068	-214. 0915885027
20000	-43. 41420269442	-239. 4267073056
30000	-53. 57500182079	-249. 2101171346
40000	-60. 92319076981	-254. 2943254827
50000	-66. 66800901692	-257. 392654312
60000	-71. 3806366196	-259. 4743567321
70000	-75. 3742659182	-260. 9679231076



100
80
60
40
20
0
ANALOG (T)
0°
50°
100°
150°
180°
200°
250°
PHASE ANGLE (DEGREES)
ϕ_m = 40°
10000
.1
1
10
100
1000
FREQUENCY (Hz)
G_m = 13 dB

CALCULATION OF LINE TRANSMISSION CHARACTERISTIC (F)

As previously derived

$$F = \frac{V_o(s)}{V_g(s)} = \frac{K\,H_e(s)}{1+T}$$

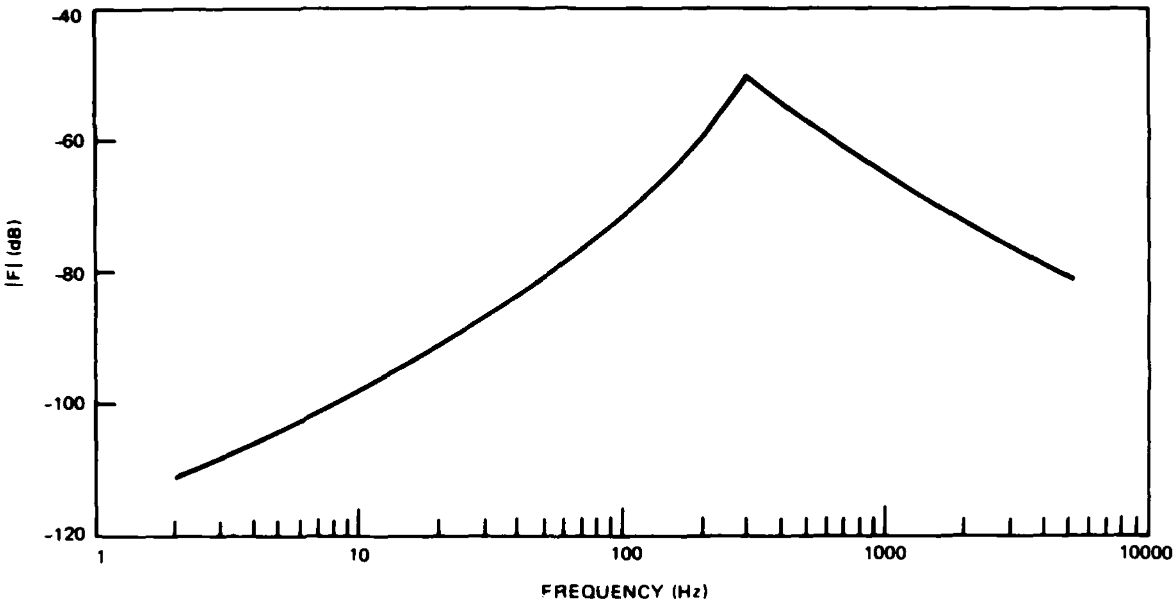
$$T \simeq \frac{4.47 \times 10^4\,(S+2500)}{(S+1)(S+530)}$$

$$\frac{1}{1+T} = \frac{(S+1)(S+530)}{S^2 + 10^5 S + 2.5 \times 10^8}$$

$$\therefore F = \frac{K\,H_e(s)}{1+T} = \frac{1.33 \times 10^5 (S+1)(S+530)}{(S+394 \pm 1920j)(S+2560)(S+97400)}$$

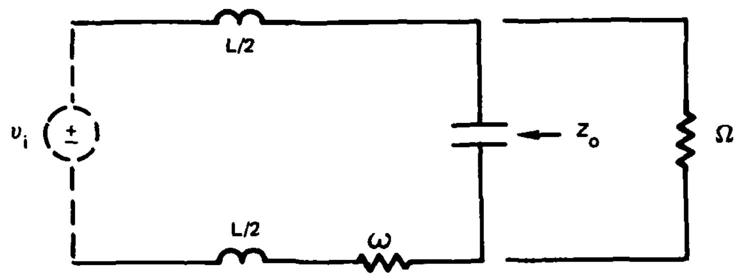
A short program to list the magnitude of F in dB follows along with the program results.

```
10 SELECT PRINT 215
20 PRINT "BODE MAGNITUDE PLOT FOR F"
30 PRINT "FREQUENCY",,"20LOG F"
40 T=.1
50 FOR V=1 TO 7
60 FO=V*T
70 W=FO*6.2831853
80 X=W^2
90 A=SQR(X+1)
100 B=SQR(X+530^2)
110 C=SQR((W+1920)^2+394^2)
120 D=SQR((W-1920)^2+394^2)
130 E=SQR(X+2664^2)
140 F=SQR(X+42000^2)
150 K=1.33E5
160 Z=8.6858896*LOG(K*A*B/(C*D*E*F))
170 PRINT FO,,Z
180 NEXT V
190 T=T*10
200 IF FO<=30000 THEN 50
210 END
BODE MAGNITUDE PLOT FOR F
FREQUENCY                                 20LOG F
 .1                                       -134.2583668737
 .2                                       -131.5886733391
 .3                                       -129.1203521346
 .4                                       -127.0602752454
 .5                                       -125.3411392965
 .6                                       -123.8812742643
 .7                                       -122.6186622233
 1                                        -119.6305403595
 2                                        -113.6891592255
 3                                        -110.1792053787
 4                                        -107.6810968523
 5                                        -105.7393707799
 6                                        -104.1497679032
 7                                        -102.8030029581
 10                                       -99.67235170605
 20                                       -93.45826228342
 30                                       -89.62770951046
 40                                       -86.72423445548
 50                                       -84.30581660682
 60                                       -82.18561991546
 70                                       -80.26996279624
 100                                      -75.31082022409
 200                                      -62.54854827245
 300                                      -52.52513022923
 400                                      -55.10086122308
 500                                      -58.58053088105
 600                                      -60.82878267158
 700                                      -62.48442685684
 1000                                     -65.92864827607
 2000                                     -72.33168895113
 3000                                     -76.28670941847
 4000                                     -79.31992983944
 5000                                     -81.85956675554
 6000                                     -84.08015911984
 7000                                     -86.06811965763
 10000                                    -91.05404498366
 20000                                    -101.9511947726
 30000                                    -108.7454156376
 40000                                    -113.6521653376
 50000                                    -117.4858868851
 60000                                    -120.6297760234
 70000                                    -123.2935008511
```



Line Transmission Characteristic

CALCULATION OF OUTPUT FILTER OUTPUT IMPEDANCE Z_o



$$Z_o = \frac{1}{C} \frac{\left(S + \frac{R_I}{L}\right)}{\left(S^2 + \frac{R_I}{L} S + \frac{1}{LC}\right)}$$

$L = 1.5 \times 10^{-5}$ H

$C = 1.76 \times 10^{-2}$ F

$R_I = 0.0016$ Ω

$$Z_{eo} = \frac{56.8(S + 107)}{(S + 53.5 + 1950j)(S + 53.5 - 1950j)}$$

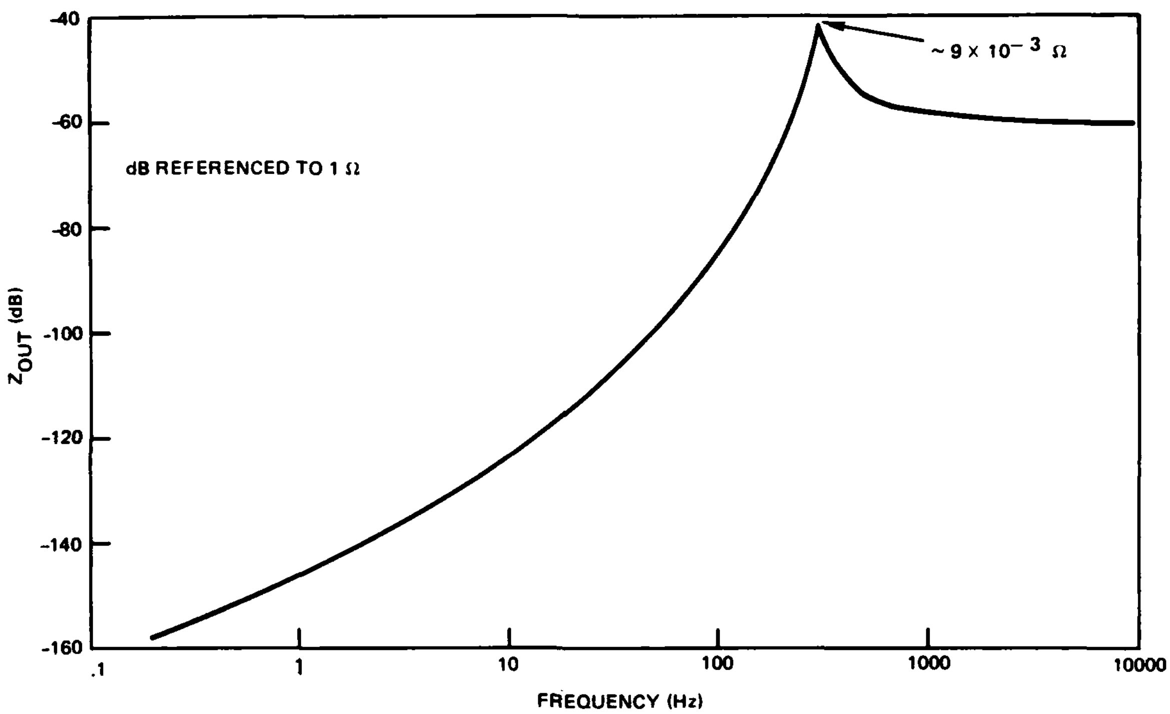
$$Z_o = \frac{Z_{eo}}{1 + T}$$

$$\frac{1}{1 + T} \simeq \frac{(S + 1)(S + 530)}{(S + 2664)(S + 42000)}$$

```
1 SELECT PRINT 215
5 PRINT
10 PRINT
20 PRINT "BODE MAGNITUDE PLOT FOR Z OUT"
30 PRINT "FREQUENCY",,"20LOG F"
35 PRINT
40 T=.1
50 FOR V=1 TO 7
60 FO=V*T
70 W=FO*6.2831853
80 X=W^2
90 A=SQR(X+1)
100 B=SQR(X+107^2)
110 C=SQR(X+530^2)
120 D=SQR((W+1950)^2+53.5^2)
130 E=SQR((W-1950)^2+53.5^2)
140 F=SQR(X+2664^2)
150 G=SQR(X+42000^2)
160 K=56.8
165 Z=8.6858896*LOG(K*A*B*C/(D*E*F*G))
170 PRINT FO,,Z
180 NEXT V
190 T=T*10
200 IF FO<=30000 THEN 50
210 END
```

BODE MAGNITUDE PLOT FOR Z OUT

FREQUENCY	20LOG F
.1	-160.9782033388
.2	-158.3080603457
.3	-155.8389901459
.4	-153.7778648801
.5	-152.0573813897
.6	-150.595869928
.7	-149.3313129116
1	-146.335569153
2	-140.3496218135
3	-136.7663913533
4	-134.1677270952
5	-132.1000409922
6	-130.3612833862
7	-128.8445975687
10	-125.1049027057
20	-116.4105991111
30	-110.2090226694
40	-105.2905910576
50	-101.1730458747
60	-97.59961293336
70	-94.41967435867
100	-86.43423027498
200	-67.03732805901
300	-40.98211093066
400	-52.50482065394
500	-55.41112069624
600	-56.38308221778
700	-56.81883989107
1000	-57.28120544429
2000	-57.7217158131
3000	-58.16390281947
4000	-58.70138208374
5000	-59.30420350322
6000	-59.94191809214
7000	-60.59139162814
10000	-62.47990940045
20000	-67 35691286579
30000	-70.62939241203
40000	-73.03739672764
50000	-74.93293159975
60000	-76.49320319837
70000	-77.81799668568



Output impedance